ECON 439

Quiz 04—Exit from a Dying Industry

Kevin Hasker

1. *(4 points)* Please read and sign the following statement:

   I promise that my answers to this test are based on my own work without reference to any notes, books, or the assistance of any other person during the test.

   Name and Surname: _

   Student ID: _

   Signature:

   _

a	b	c	$\underline{t}$	$\bar{t}_1$	$\bar{t}_2$	$\pi_2^{\bar{t}_1+}$	$\pi_2^{\bar{t}_1+1+}$
24	10	4	10	14	20	44	60
20	10	5	5	10	15	25	50
25	11	6	8	14	19	24	60
30	13	8	9	17	22	16	80

2. *(16 points total)* Consider an industry which has the demand curve $P(t, Q) = a - t - Q$, where $t \in \{0, 1, 2, ...\}$ is the time period and Q is the total output. There are two firms in this industry, their strategy is simply the last period in which they produce—$t_i \in \{0, 1, 2, ...\}$. In period t, if $t \leq t_i$ they produce k_i ($k_1 = b$, $k_2 = c$) if $t > t_i$ they produce no output. These firms have no cost of production, thus their profit in period t is their revenue: $P(t, Q_t) q_{it}$ where $q_{it} = 0$ if $t > t_i$ and k_i if $t \leq t_i$. Their objective is to maximize the sum of their profits in all periods.

   Assume a firm will choose to produce if indifferent. And please notice that for simplicity price can be negative.

   (a) *(6 points)* Let $\underline{t}$ be the last period in which if both firms produce they will make at least zero profit, $\bar{t}_i$ be the last period in which firm i produces alone then they can make at least zero profit. Find $(\underline{t}, \bar{t}_1, \bar{t}_2)$ note that $\bar{t}_2 > \bar{t}_1 > \underline{t}$.

   Solution 1 *Clearly what matters is the last period when price will be at least zero.*

   If both produce $Q = b + c$ thus

$$\begin{aligned} 0 &= P = a - \underline{t} - b - c \\ \underline{t} &= a - b - c \end{aligned}$$

   likewise if only firm one produces then

$$\begin{aligned} 0 &= P = a - \bar{t}_1 - b \\ \bar{t}_1 &= a - b \end{aligned}$$

   and therefore $\bar{t}_2 = a - c$.

(b) *(4 points)* Prove that $t_i \in \{\underline{t}, \underline{t}+1, \underline{t}+2, ..., \bar{t}_i\}$ for both i.

Solution 2 *If $t_i < \underline{t}$ then a firm can make a weakly positive profit in the periods between t_i and $\underline{t}$, thus they will produce by assumption. If $t_i > \bar{t}_i$ they must be strictly losing money between $\bar{t}_i$ and t_i, thus they should choose at least $\bar{t}_i$.*

(c) *(2 points)* Assuming that $\min\{t_1, t_2\} \geq \bar{t}_1 - 1$, find the unique equilibrium values of (t_1, t_2).

Solution 3 *From this point on, points should only be given for work towards the answer, not simply stating the answer.*
If firm 2 choose $t_2 \geq \bar{t}_1$ then they should choose $t_2 = \bar{t}_2$. In the periods $\bar{t}_1 + 1$ to $\bar{t}_2$ they will make:

$$\begin{aligned}
\pi_2^{\bar{t}_1+1+} &= \sum_{t=\bar{t}_1+1}^{\bar{t}_2} (a-t-c)\,c = c\left(\sum_{t=\bar{t}_1+1}^{\bar{t}_2} (a-t-c)\right) \\
&= c\left(\sum_{t=a-b+1}^{a-c} (a-t-c)\right) \\
&= \frac{1}{2}c\left(b^2 - 2bc - b + c^2 + c\right)
\end{aligned}$$

If firm 1 chooses to produce in period $\bar{t}_1$firm 2 will loose

$$(a-(a-b)-b-c)\,c = -c^2$$

Thus their profit form choosing $t_2 = \bar{t}_2$ is at least:

$$\underline{\pi}_2^{\bar{t}_1+} = -c^2 + \frac{1}{2}c\left(b^2 - 2bc - b + c^2 + c\right)$$

which is strictly positive. Thus they have a dominant strategy of $t_2 = \bar{t}_2$. The best response of firm one is to choose $t_1 = \bar{t}_1 - 1$, or to not produce this period.

(d) *(2 points)* Given the proof in the last step, assuming that $\min\{t_1, t_2\} \geq \bar{t}_1 - 2$, find the unique equilibrium values of (t_1, t_2).

Solution 4 *From the last step we know that $t_1 \leq \bar{t}_1 - 1$. Thus if firm 2 chooses $t_2 = \bar{t}_2$ their profit from next period on is*

$$c\left(\sum_{t=a-b}^{a-c} (a-t-c)\right) = \frac{1}{2}c\left(b^2 - 2bc + b + c^2 - c\right)$$

which is higher than before, the worst they can make in this period is:

$$(a-(a-b-1)-b-c)\,c = -c\,(c-1)$$

which is lower than before, thus they will choose $t_2 = \bar{t}_2$, and thus firm one will choose $t_1 = \bar{t}_1 - 2$.

(e) *(2 points)* Find the unique equilibrium of the entire game, proving your conclusion.

Solution 5 *The equilibrium will be* $t_2 = \bar{t}_2$, $t_1 = \underline{t}$, *the points will be given for the argument. We proceed by iteration, assume that* $\min\{t_1, t_2\} \geq t - 1$ *and* $t_1 \leq t$ *for* $\bar{t}_1 - 2 > t > \underline{t}$. *The profits firm 2 gets in periods* $\bar{t}_1 + 1$ *to* $\bar{t}_2$ *is high enough for them to be willing to take the loss in period* $\bar{t}_1$, *the loss in any period* $t \leq \bar{t}_1$ *are lower than that, thus it is always optimal to choose* $t_2 = \bar{t}_2$, $t_1 = t - 1$. *Thus by iteration we are done.*